EE 505

Laboratory Experiment 1

Spring 2025

Statistical Characterization of Matching-Critical Circuits

Most data converters depend upon the matching of some critical components with higher resolution data converters invariably requiring more stringent matching requirements. Though calibration is often used to reduce matching requirements, matching is still important in many data converters.   Invariably a statistical analysis or statistical simulations are required to accurately predict the performance of matching-critical circuits.

In this experiment, matching requirements in R-2R networks will be considered and experimental validation will be based upon using discrete components. The same concepts are applicable to integrated components but discrete components will be used to facilitate experimental measurements early in the course.

The R-2R Network

What may appear to be two variants of a 5-bit R-2R network are shown in Fig.1. (These are actually identical structures, only the labeling of the components has changed). Though only depicted as a 5-bit network, the resolution can be increased by including additional bit slices denoted in the tan shaded areas. One of the attractive properties of the R-2R network is that the number of elements increases linearly with the number of bits of resolution.
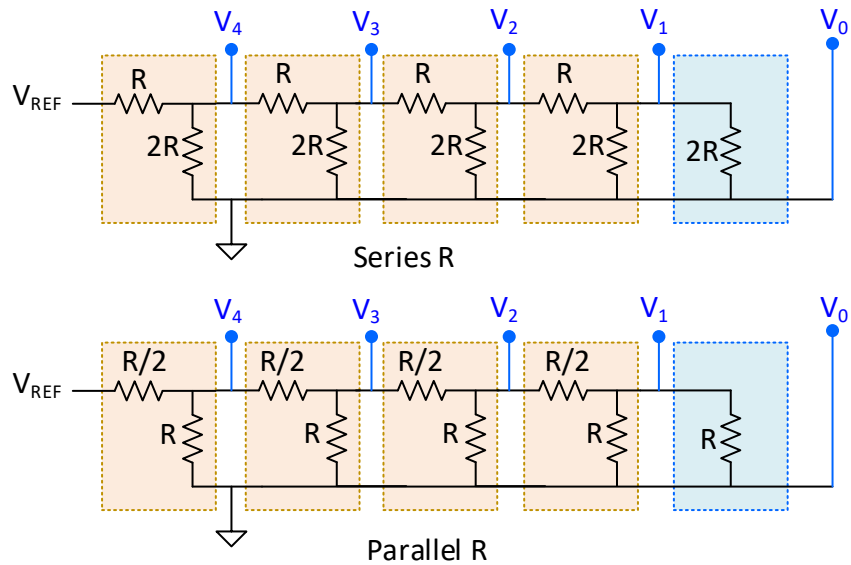

Fig. 1   R-2R Networks

The R-2R networks are characterized with resistors in the shunt portion of a bit-slice being double the resistance value of resistors in the series portion of the bit-slice. The R-2R networks are terminated on the right with a single resistor equal in value to the shunt resistors in the bit-slice.

There are several variants in how the R-2R network is used in a data converter. One way to form a DAC is to create a weighted sum of the tap voltages designated as V_0, $V_1,\ldots V_4$ in the 5-bit DAC. The weights are either 0 or 1. Switches that connect these tap voltages to the summer and the summer needed to complete the DAC are not shown in the figure. Of course, the overall performance of such a DAC would depend on both the performance of the R-2R network and the performance of the circuit that creates the weighted sum.

A unit resistor is often used to form an R-2R network. If a unit resistor of value R is used, the circuits of Fig. 1 represent two ways to create the R-2R network with a unit resistor. One is termed the Series-R and the other the Parallel-R configuration. In the Series-R configuration, two unit resistors are placed in series to realize the shunt elements in the bit-slice. In the Parallel-R configuration, two unit resistors are placed in parallel to realize the series elements in the bit-slice.

A second variant of using a R-2R network to create a DAC is shown in Fig. 2 using the Series R structure of Fig. 1. Of course, the Parallel R structure could be used here as well. In this structure, each of the "2R" resistors is connected to either V_{REF} or ground depending upon the Boolean input <b_3 b_2 b_1 b_0>. This structure eliminates the need of a summing network but introduces switch impedances in the R-2R structure itself which, if not ideal, will affect performance.
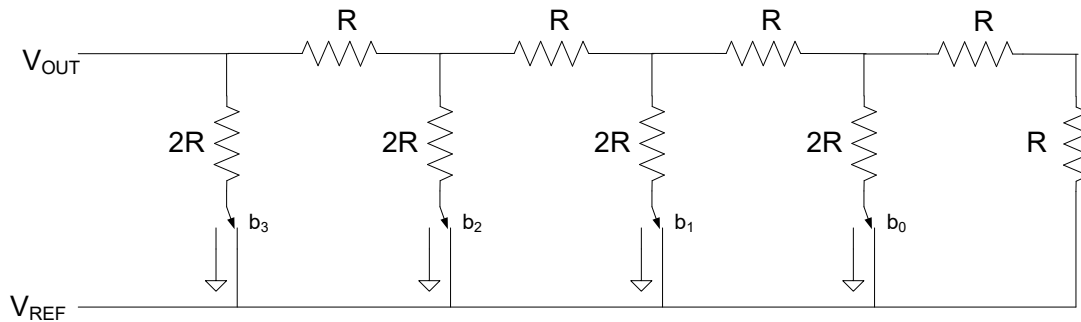

Fig. 2 R-2R DAC Using Switched Excitation

In this experiment, we will attempt to answer the following two questions:

Question 1: How does the performance of an R-2R DAC depend upon the statistical variation of the components

Question 2: For a given total area, will the Series-R or the Parallel-R structure provide a higher yield if the only yield loss is due to random variations in the resistor values?

The following question is also of interest but will not be addressed at this time to limit the scope of this experiment.

Question 3: If the area is allocated so that the yield of a Series-R and a Parallel-R structure is the same, what increase in area is required if the non-optimal implementation is used instead of the optimal area implementation.

Before addressing these questions, a statistical model of the resistors is needed.

Characterization of Resistors

Resistance Measurement

       Measure the resistance value of the 25 resistors that were used in the DAC in the previous part of the experiment using the Keysight 34470A Multimeter. When making these measurements, place all 25 resistors on a protoboard and number these resistors R_1 through R_{30}. Keep track of the numbering of these resistors and the corresponding value you measured for each resistor throughout the remainder of this experiment. A convenient way to make this measurement might be to place the resistors in the protoboard as depicted below.
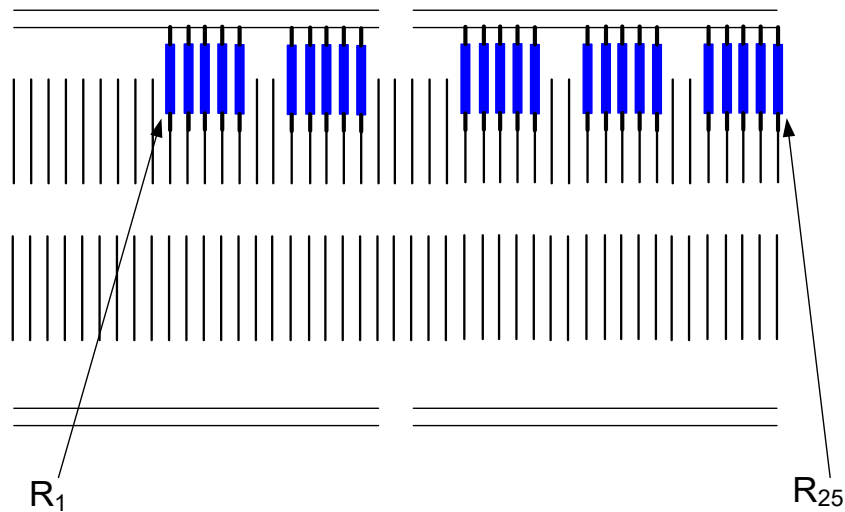
From these measurements determine the mean, the standard deviation and plot the distribution of the resistors. Comment on what you observe from these measurements.

Measurement Accuracy

       Determine how accurate the measurements you made are. Specify the accuracy that you think you have obtained in percent, in parts per million (PPM), and in

terms of the number of bits. The accuracy in PPM relates to the accuracy in percent by $X_{PPM} = 10^4X$ where X is the accuracy in percent. The definition of the accuracy in "bits" is based upon the relationship

$$\frac{1}{2^n} = \frac{X}{100} \qquad (1)$$

where X is the accuracy in % and n is the number of bits. To save you the calculation, the relationship between n and X is given explicitly as

$$n = \frac{2-\log_{10}X}{\log_{10}2} \qquad (2)$$

Thus, 1% accuracy corresponds to about 6.5 bits and 10000 PPM, 0.1% accuracy to about 10 bits and 1000 PPM, .01% accuracy to about 13 bits and 100 PPM and .001% accuracy to a little better than 16 bits and 10 PPM.

When making this determination, consider both the absolute and relative accuracy you think you have achieved with your measurements. There are several factors that affect the accuracy of your measurement. These include the accuracy of the test equipment, the temperature at which the device was measured, and the effects of any contact, lead, and circuit board resistance. For your information, the datasheet for the Keysight 34470A is linked on the class WEB site adjacent to this laboratory experiment. Be quantitative in your assessment of the accuracy you think you have obtained.

Since we have used the term "accuracy" in this assessment, be rigorous on your definition of accuracy. You will notice that the Keysight 34470A, as well as other competing instruments, will give an accuracy specification. How is the accuracy in these specifications defined?

Temperature Coefficient

Determine the temperature coefficient in PPM/ºC for the resistors (measurement on a single resistor should be adequate). From this measurement, how stable must the temperature of the resistor be to maintain the accuracy you determined in the previous part of this experiment? The oven in the laboratory should be useful for measuring the temperature coefficient.


Component Drift

Monitor the value of one of your resistors for a period of 10 minutes taking measurements approximately every 30 seconds. How does this value compare to what you obtained in the previous measurement. Are the changes, if any, consistent with the accuracy you determined in the measurements?

Contact Resistance

Contact resistance is a major problem in accurately measuring the value of a resistor. Contact resistance is associated with the resistance of the contacts to the device under test

or, in an IC, due to the contacts or vias that are needed to make a connection to the resistive region. In this experiment the contact resistance is relatively small compared to the resistors you are measuring but in integrated circuits the contact resistances can be several percent or more of the total resistance. Unfortunately the contact resistance is often highly variable. Circuits are often designed to be insensitive to contact resistance and the techniques used to make a circuit insensitive to contact resistance are the same as are used to make contact-insensitive measurements of resistors in the laboratory. This is based upon what are often termed 4-point measurements (alternatively termed force and sense or Kelvin sensing). A 4-point measurement circuit is shown below. In this circuit a current is forced through the resistor and the voltage
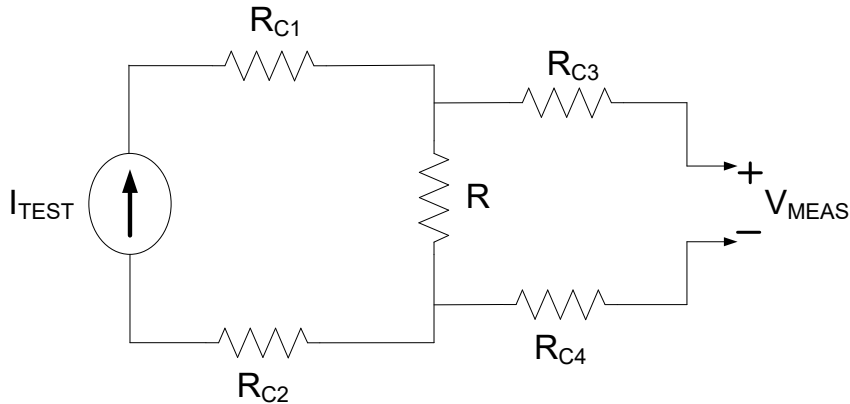
is measured across the resistor terminals. In contrast to the two-point measurements discussed previously, there are 4 contact resistances present in this circuit designated as $R_{C1} - R_{C4}$. However. since no current flows through the contacts connecting the voltmeter, none of the contact resistances affect the measured resistance.

Develop a method for measuring the contact resistance associated with inserting components into the protoboards used in the laboratory. **Measure the contact resistance associated with the protoboards.**

4-Bit DAC Design

Build a 4-bit DAC on a protoboard using the R-2R DAC architecture of Fig. 2 and the resistors you measured previously. This figure is repeated in Fig. 3. Use wires for the switches. Use resistors R_1 and R_2 for the left-most "2R resistor", use R_3 for the left-most series resistor, use R_4 and R_5 for the next "2R" resistor etc. as indicated by the resistor numberings in Fig. 3.
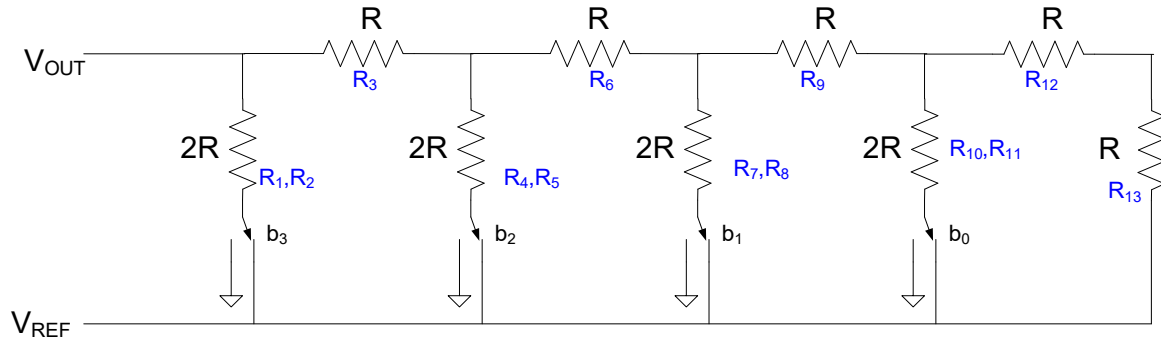
Fig. 3   Implementation of R-2R DAC

Measure the transfer characteristics of the DAC and determine the INL and DNL of this structure (use the standard end-point fit line to determine the INL). Compare the measured INL with that you obtain with a Spectre simulation using the measured values for R_1-R_{13}. Neglect contact resistances in this simulation. Determine the ENOB of your DAC (use the INL ENOB definition).

In Excel, sort the value of the resistors R_1-R_{13} from the largest to the smallest. Designate these as R_{1S}-R_{13S}. After sorting, replace R_1 in your DAC with R_{6S}, Replace R_2 in your DAC with R_{8S}, Replace R_3 in your DAC with R_{7S}, Replace R_4 in your DAC with R_{5S} and Replace R_5 in your DAC with R_{9S}. By doing these replacements by doing a sequence of interchanges, you will still have a complete DAC. By simulation, predict the performance of the modified DAC. Measure the transfer characteristics of the modified DAC and determine the INL and DNL of this structure and determine the ENOB of your DAC (use the INL ENOB definition). Compare the performance of the modified DAC with what was obtained using the same 13 resistors that were randomly selected to form the DAC.

There are different clever strategies for inter-changing resistors in a circuit and if interchanging matching-critical components improves performance, this becomes a calibration scheme that can compensate for undesired mismatch effects.

For reference purposes, the ideal transfer characteristics and the transfer characteristics of a nonideal DAC are depicted below.
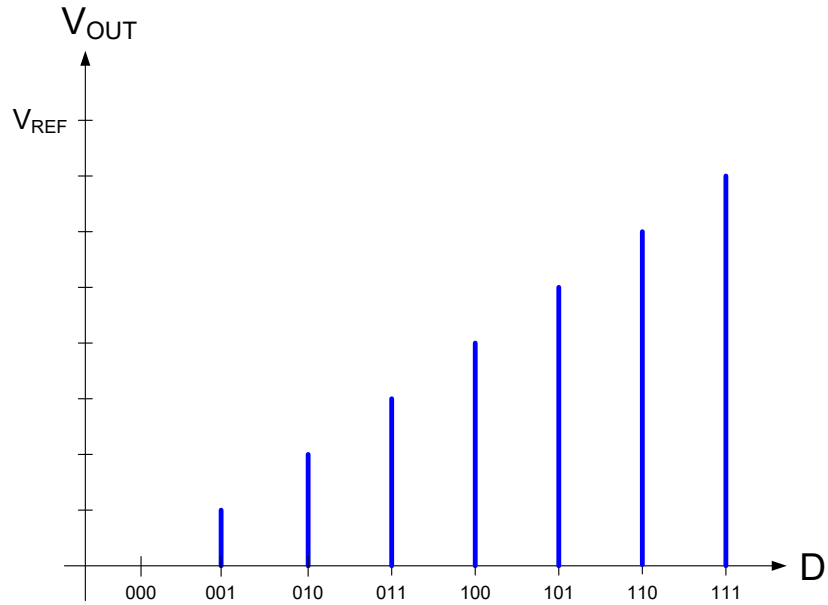
Transfer Characteristics of Ideal DAC
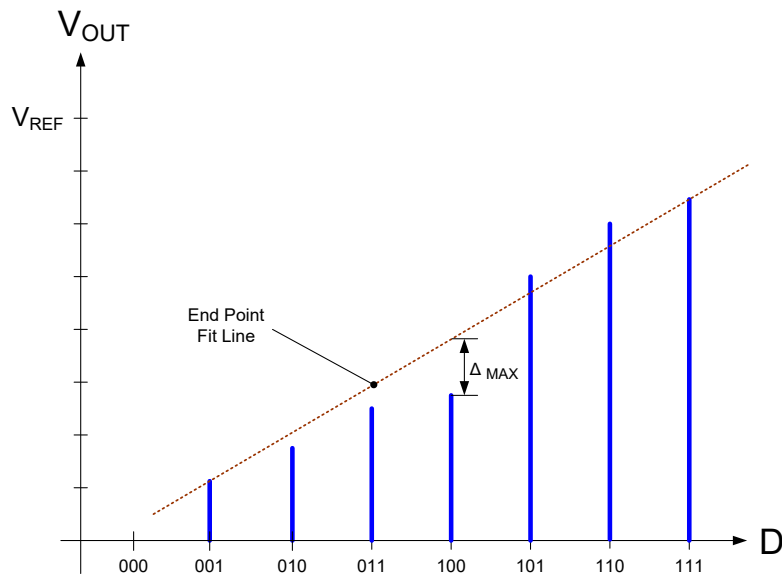
Transfer Characteristics of Actual DAC

Series R or Parallel R Structures

Determine whether the Series-R and Parallel-R implementation of a 4-bit DAC has better performance for a given total area (that is the total area is the same).   At this point, make this determination by computer simulation. Model your devices in this simulation based upon the Pelgrom matching parameter, A_ρ . Since we do not have the Pelgrom parameter for the resistors we are using in this experiment, use the measured

standard deviation from the 25 resistors to obtain an estimate of the Pelgrom matching parameter. Assume the normalized standard deviation is given by the expression

$$\sigma_{\frac{R_{ran}}{R_{nom}}} = \frac{A_\rho}{\sqrt{A_R}} \, .$$

where the area of the resistor is A_R. Note that the number of resistors in the Series-R and the Parallel-R implementations is not the same so for making area comparisons, this difference should be taken into account. Since these resistors are discrete thin-film resistors, the actual area is not known but correspondingly the value for A_ρ that you will extract from measurements will depend upon the area of your test devices. For convenience, assume the area of your test devices , A_R , are all $100u^2$ (this assumption is not necessary but is used strictly for convenience).

Test Environment Validation

When testing data converters, it is general practice to expect the accuracy of the test equipment to be about a factor of 10 (i.e. approximately 3-bits) better than the required accuracy of the device that is being tested. The Keysight 34470A is termed a 7 ½ digit multimeter where the factor of "7.5" refers to decimal digits, not binary digits.

a) If it truly has 7.5 digit performance, what would be the performance of the multimeter in binary bits?

b) Actually the ½ digit does not refer to a full half of a decimal digit but rather to the ability to have either a 0 or a 1 in the MSB decimal column. With this understanding, what is the number of binary bits of resolution for the multimeter if the number of decimal digits truly reflects the performance?

c) Based upon the specifications of the multimeter, how many binary digits of performance does this device really provide when the input is a full-scale voltage?

d) If the Keysight 34470A is used to test the pseudo-static linearity performance of data converters, how many bit converters can it test if a) V_{REF}=4V and b) if V_{REF}=1V.

Appendix

Derivation of Standard Deviation of Simple Circuit

Since a statistical analysis is required as a part of this experiment, a simple circuit will be considered in this Appendix that may serve as a refresher on statistical analysis. Consider the simple two-transistor inverter shown in Fig. A1.
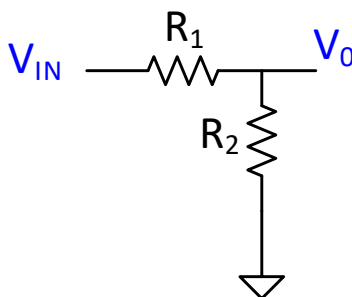

Fig. A1 Voltage Divider Circuit

The resistors R_1 and R_2 are assumed to have a nominal part and a random part and thus can be expressed as

$$R_1 = R_{1N} + R_{1R}$$
$$R_2 = R_{2N} + R_{2R} \qquad (A1)$$

In what follows, the standard deviation of the attenuation factor will be derived under two separate assumptions. One assumption is that the resistors are made with discrete components and the other assumption is that they are thin-film integrated components coming from the same thin-film layer. For the integrated structures, it will be assumed that the random part of the resistance is due totally to local random variations (i.e. gradient effects will be assumed to be absent either through a process with very small gradients or by the use of common-centroid layouts to cancel gradient effects). In both cases, it will be assumed that the random part of the resistors are uncorrelated.

The attenuation factor can be expressed as

$$\frac{V_0}{V_{IN}} = \frac{R_2}{R_1 + R_2}$$

Substituting from (A1) we obtain

$$\frac{V_0}{V_{IN}} = \frac{R_{2N} + R_{2R}}{R_{1N} + R_{2N} + R_{1R} + R_{2R}}$$

This can be rewritten after some routine manipulations as

$$\frac{V_0}{V_{IN}} = \frac{R_{2N}}{R_{1N} + R_{2N}} \bullet \frac{1 + \dfrac{R_{2R}}{R_{2N}}}{1 + \dfrac{R_{1R}}{R_{1N} + R_{2N}} + \dfrac{R_{2R}}{R_{1N} + R_{2N}}}$$

Unfortunately, this is a nonlinear function of the random variables R_{1R} and R_{2R}. But since the random parts are small relative to the nominal parts of the resistors, the term

$$\frac{1}{1 + \dfrac{R_{1R}}{R_{1N} + R_{2N}} + \dfrac{R_{2R}}{R_{1N} + R_{2N}}}$$

Can be expanded in a Taylor's series truncated after first-order terms to obtain

$$\frac{V_0}{V_{IN}} = \frac{R_{2N}}{R_{1N} + R_{2N}} \bullet \left(1 + \frac{R_{2R}}{R_{2N}}\right)\left(1 - \frac{R_{1R}}{R_{1N} + R_{2N}} - \frac{R_{2R}}{R_{1N} + R_{2N}}\right)$$

Neglecting product terms of two random variables which should be very small, after some routine manipulations we obtain

$$\frac{V_0}{V_{IN}} = \frac{R_{2N}}{R_{1N} + R_{2N}} \bullet \left(1 + \frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N} + R_{2N}} - \frac{R_{2R}}{R_{1N} + R_{2N}}\right)$$

This can be manipulated to obtain

$$\frac{V_0}{V_{IN}} = \frac{R_{2N}}{R_{1N} + R_{2N}} \bullet \left(1 - \frac{R_{1N}}{R_{1N} + R_{2N}}\frac{R_{1R}}{R_{1N}} + \frac{R_{1N}}{R_{1N} + R_{2N}}\frac{R_{2R}}{R_{2N}}\right)$$

This is now the weighted sum of two uncorrelated random variables so it follows that the variance is given by the expression

$$\sigma^2_{\frac{V_0}{V_{IN}}} = \left(\frac{R_{2N}}{R_{1N} + R_{2N}}\right)^2 \left(\frac{R_{1N}}{R_{1N} + R_{2N}}\right)^2 \left(\sigma^2_{\frac{R_{1R}}{R_{1N}}} + \sigma^2_{\frac{R_{2R}}{R_{2N}}}\right) \qquad \text{(A2)}$$

Equation (A2) applies for either discrete or integrated resistors.

If the resistors are integrated, it follows that

$$\sigma^2_{\frac{R_{1R}}{R_{1N}}} = \frac{A^2_\rho}{A_{R1}}$$

$$\sigma^2_{\frac{R_{2R}}{R_{2N}}} = \frac{A^2_\rho}{A_{R2}}$$

where A_ρ is the Pelgom matching parameter and A_{R1} and A_{R2} are the areas of resistors R_1 and R_2 respectively. Substituting back into (A2) we obtain

$$\sigma^2_{\frac{V_0}{V_{IN}}} = \left(\frac{R_{2N}}{R_{1N} + R_{2N}}\right)^2 \left(\frac{R_{1N}}{R_{1N} + R_{2N}}\right)^2 A^2_\rho \left(\frac{1}{A_{R1}} + \frac{1}{A_{R2}}\right)$$

In the special case where $R_{1N} = R_{2N}$, this reduces to

$$\sigma_{\frac{V_0}{V_{IN}}} = \frac{1}{2}\frac{A_\rho}{\sqrt{A_T}}$$

where A_T is the total resistor area.

With discrete resistors, if they are the are the same type of resistors and $R_{1N}=R_{2N}$, it follows directly from (A1) that

$$\sigma_{\frac{V_0}{V_{IN}}} = \frac{1}{\sqrt{8}}\left(\sigma_{\frac{R_{1R}}{R_{1N}}}\right)$$

Statistical Analysis with Sensitivity Functions

The previous statistical analysis was an ad hoc approach but was presented to be intuitive. It was also quite tedious and becomes more tedious for more intricate nonlinear functions of random variables. A much more systematic approach that is particularly useful for linearizing nonlinear random functions is based upon a standard sensitivity analysis. The approach will provide the same results, often with considerably less effort. The sensitivity-based statistical analysis will now be formalized.

Assume Y is a function of n uncorrelated random variables $\{x_1, x_2, \ldots x_n\}$ where the random variables themselves can be expressed as

$$x_i = x_{iN} + x_{iR}$$

where x_{iN} is the nominal part of x_i and x_{iR} is the random part of x_i. For short-hand notation, denote X, X_R, and X_N as the vectors

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} \qquad X_N = \begin{bmatrix} x_{1N} \\ x_{2N} \\ \ldots \\ x_{nN} \end{bmatrix} \qquad X_R = \begin{bmatrix} x_{1R} \\ x_{2R} \\ \ldots \\ x_{nR} \end{bmatrix}$$

This function Y can be expressed as

$$Y = f\left(x_{1R}, x_{2R}, \ldots x_{nR}\right)$$

Expanding Y in a power series and truncating after first-order terms we obtain

$$Y \cong Y_N + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial x_i}{\partial x_{iR}}\bigg|_{X_R=0} x_{iR}$$

Note that Y is now a linear sum of uncorrelated random variables.
But for all i,

$$\frac{\partial x_i}{\partial x_{iR}} = 1$$

Thus

$$Y \cong Y_N + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\bigg|_{X_i=X_N} x_{iR}$$

This can be rewritten as

$$Y \cong Y_N + f\big|_{X_i=X_N} \sum_{i=1}^{n} \frac{x_{iN}}{f} \frac{\partial f}{\partial x_i}\bigg|_{X_i=X_N} \frac{x_{iR}}{x_{iN}}$$

But the argument in the summation is recognized as the standard sensitivity function, thus

Page - 11 -

$$Y \cong Y_N + Y_N \sum_{i=1}^{n} S_{x_i}^{f}\Big|_{X_i=X_N} \frac{x_{iR}}{x_{iN}}$$

The variance of Y can thus be expressed as

$$\sigma_Y^2 \cong Y_N^2 \sum_{i=1}^{n} \left(S_{x_i}^{f}\Big|_{X_i=X_N} \right)^2 \sigma_{\frac{x_{iR}}{x_{iN}}}^2$$

This is the key result.

Now, we will reconsider the previous simple attenuator example

$$Y = \frac{V_0}{V_{IN}} = \frac{R_2}{R_1 + R_2}$$

The two sensitivity functions are

$$S_{R_1}^{f} = \frac{-R_2}{\left(R_1 + R_2\right)^2} \frac{R_1}{Y}$$

$$S_{R_2}^{f} = \frac{R_1}{\left(R_1 + R_2\right)^2} \frac{R_2}{Y}$$

Thus, when evaluated at $X_i=X_N$, we obtain

$$\sigma_Y^2 \cong \left(\frac{-R_{1N}R_{2N}}{\left(R_{1N} + R_{2N}\right)^2} \right)^2 \sigma_{\frac{x_{1R}}{x_{1N}}}^2 + \left(\frac{R_{1N}R_{2N}}{\left(R_{1N} + R_{2N}\right)^2} \right)^2 \sigma_{\frac{x_{2R}}{x_{2N}}}^2$$

or equivalently as

$$\sigma_Y^2 \cong \left(\frac{R_{1N}R_{2N}}{\left(R_{1N} + R_{2N}\right)^2} \right)^2 \left(\sigma_{\frac{x_{1R}}{x_{1N}}}^2 + \sigma_{\frac{x_{2R}}{x_{2N}}}^2 \right)$$

Note this is the same as (A1).